How Fast Can You Grow?

Investigating Exponential Functions

Essential Questions

- How can we determine if an equation represents an exponential function?
- How can we determine if a data table represents an exponential function?
- How are exponential functions used in the real world?

Common Core Standards

- F.IF.7- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  - *e. Graph exponential and logarithmic functions.*

Common Core Standards

- F.LE.1- Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - *c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another*

Common Core Standards

- F.LE.2- construct linear and exponential functions, given a graph, a description of a relationship, or two input-output pairs

Common Core Standards

- F.LE.3 observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function

Warm Up 1/16/18

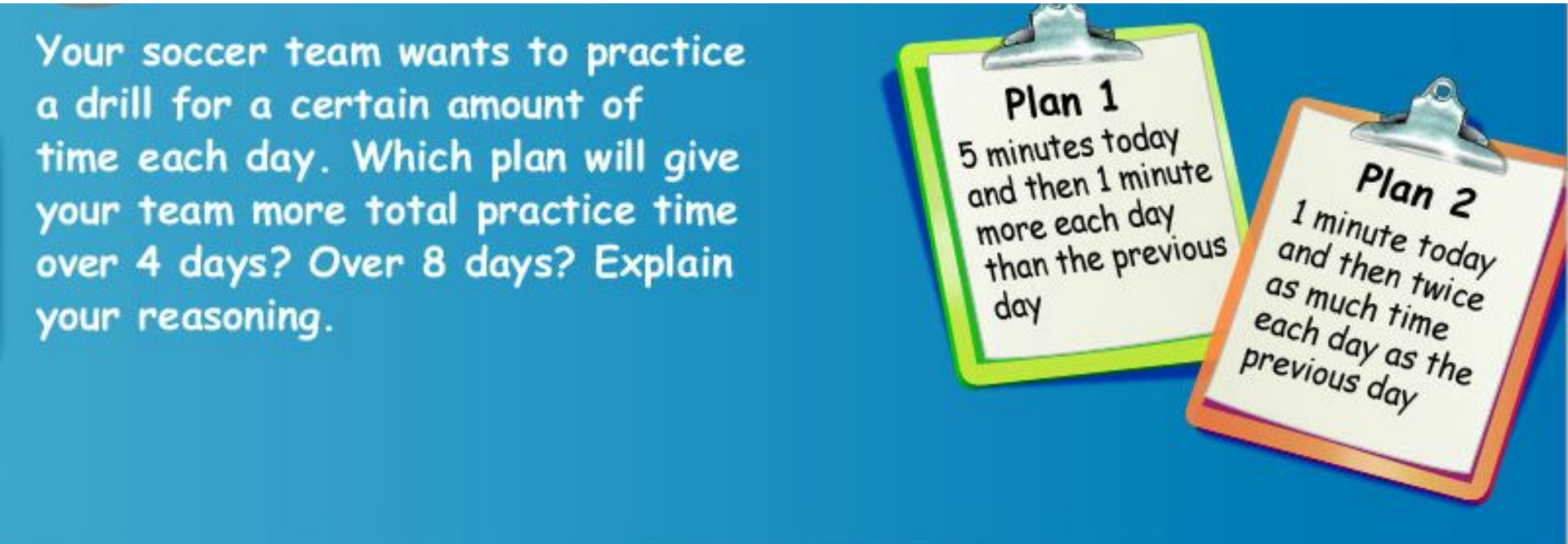
- Write 3,405,000,000,000 in scientific notation
-
- Write 2.19 x 10^9 in standard form

- Multiply: (4.6 x 10^3)(5.8 x 10^-5)

- Divide: $\frac{1.3 \times 10^{4}}{8.07 \times 10^{10}}$

5-Min Discussion

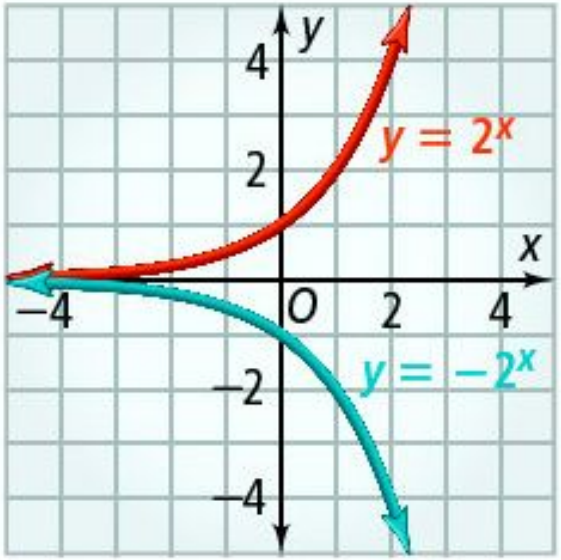
- Determine a strategy to calculate the practice times over 8 days and answer the questions
- Birth Months: Even (Plan 1)/Odd (Plan 2)

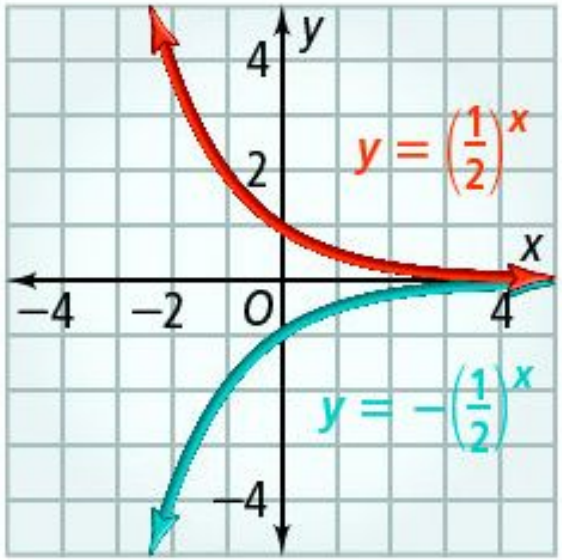


Definition

- An **exponential function** is a function of the form $y = a \cdot b^x$, where $a \neq 0$, $b > 0$, $b \neq 1$ and x is a real number.

Examples





Where Are They Used?

- Mortgage loans (home economics)
- Student loans (home economics)
- Population growth (government)
- Bacteria or virus growth (science)
- Depreciation on a vehicle (home economics)
- Investment accounts (home economics)
- Carbon dating (science)

Identify an Exponential Function

- If all of the x-values in a table of values has a **constant difference** and all of the y-values have a **constant ratio(factor)**, then the table represents an **exponential function**.

x	0	1	2	3
y	-1	-3	-9	-27

Quick Poll: Got It?

EXAMPLES:

1. Is $y = 3x^2$ an exponential function? Why or why not? Be prepared to justify your answer.

2. Does the table below represent an exponential function? Why or why not? Be prepared to justify your answer.

x	1	2	3	4
y	-1	1	3	5

Got It?

EXAMPLES: Solutions

1. NO! Not in correct form, $y = a \cdot b^x$

2. NO! Y-value's aren't changing at a constant ratio

x	1	2	3	4
y	-1	1	3	5

Graphing Exponential Functions

EXAMPLES

Graph each equation into the Y= editor:

3. $y = 3^x$

4. $y = 4\left(\frac{1}{2}\right)^x$

I Do – You Watch

Writing a Function Rule from a Table

5.Look at the table:

x	-2	-1	0	1	2
y	2	4	8	16	32

SOLUTION

STEP 1

Tell whether the function is exponential.

Writing a Function Rule from a Table

+1 +1 +1 +1

x	–2	–1	0	1	2
y	2	4	8	16	32

× 2 × 2 × 2 × 2

Here, the y-values are multiplied by 2 for each increase of 1 in x, so the table represents an exponential function of the form $y = ab^x$ where $b = 2$.

Writing a Function Rule from a Table

***STEP** 2*

Find the value of a **by finding the value of** y **when** $x = 0$.

- **The value of** y **when** $x = 0$ is 8, **so** $a = 8$.

***STEP** 3*

Write the function rule. A rule for the function is $y = 8 \cdot 2^x$.

We Do

6. **Write a rule for the function.**

x	-2	-1	0	1	2
y	3	9	27	81	243

ANSWER

Remember:

1) Find the difference in x.
2) Find the factor in y (b).
3) Find a (where x = 0).
4) Write the exponential function.

You Do – I Watch

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

SOLUTION

What is another term we can use to represent x?

2-min Review

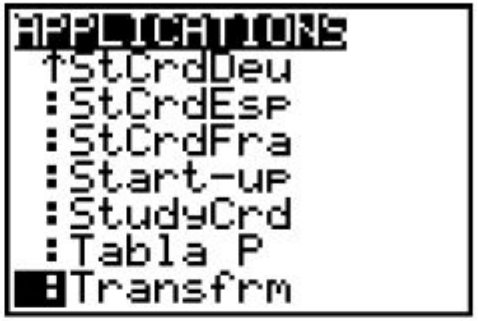
- Discuss similarities and differences between linear and exponential functions
- Discuss how exponential functions are used in the real-world
- Identify whether a set of data is an exponential function
- Practice graphing exponential functions by hand and with a calculator
- Write a function rule given a data table

Calculator Investigation

- Go to APPS
- Select: Transformation Graphing APP

Investigating the Effect of *A* and *B* on the Graph of $Y = AB^X$

1. Press [APPS] and select **Transfrm**.



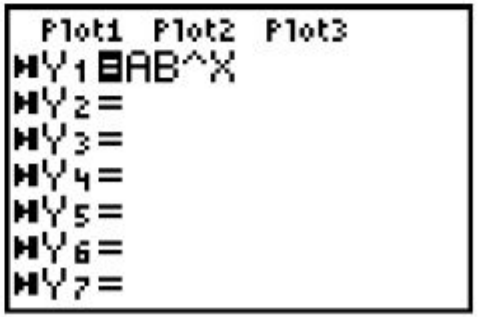
2. Press any key (except [2nd] or [ALPHA]) to start the Transformation Graphing App.

   Note: *If you do not see the screen shown, select* **Continue**.

3. In **Func** mode, press [Y=] to display the **Y=** editor. Clear any functions that are listed, and turn off any plots.

4. At **Y1=**, enter **AB^X**. Press [ALPHA] **A** [ALPHA] **B** [^] [X,T,Θ,n].

   If Play-Pause mode (>||) is not selected at the left of **Y1**, press [◄] until the cursor is over the symbol; then press [ENTER] until the correct symbol is selected.



Practice with a Partner

1. The graph appears to be a line. Why? Explain your answer.
2. If B remains 1 and A changes, what do you think will happen to the graph?
   - Make a hypothesis, and then use the arrow keys to see what happens.
3. How did the graph change?

Exponential Functions

Growth and Decay

Warm Up 1/23/18

Write the percent as a decimal.

1. 4% 2. 0.5% 3. 13.8% 4. 145%

5.

x	-2	-1	0	1	2
y	1/3	1			

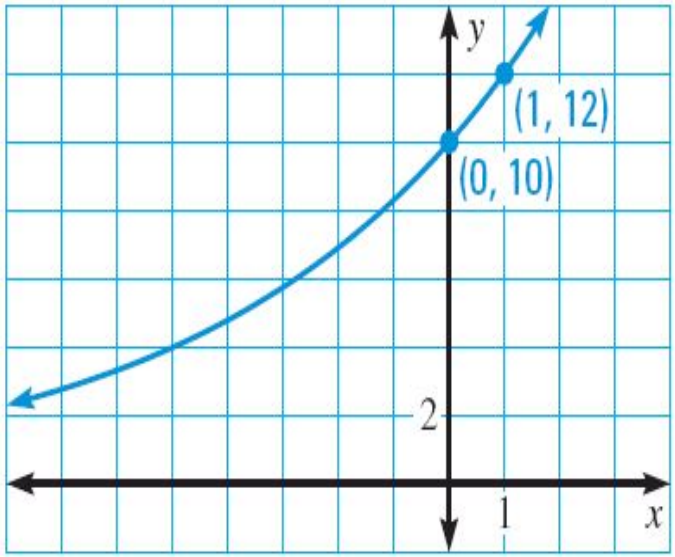
SOLUTION

***y* =**

What is another term we can use to represent x?

I Do – You Watch

1. **Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.**



SOLUTION

The graph represents exponential growth ($y = ab^x$ where $b > 1$). The y-intercept is 10, so $a = 10$. Find the value of b by using the point (1, 12) and $a = 10$.

$y = ab^x$ **Write function.**

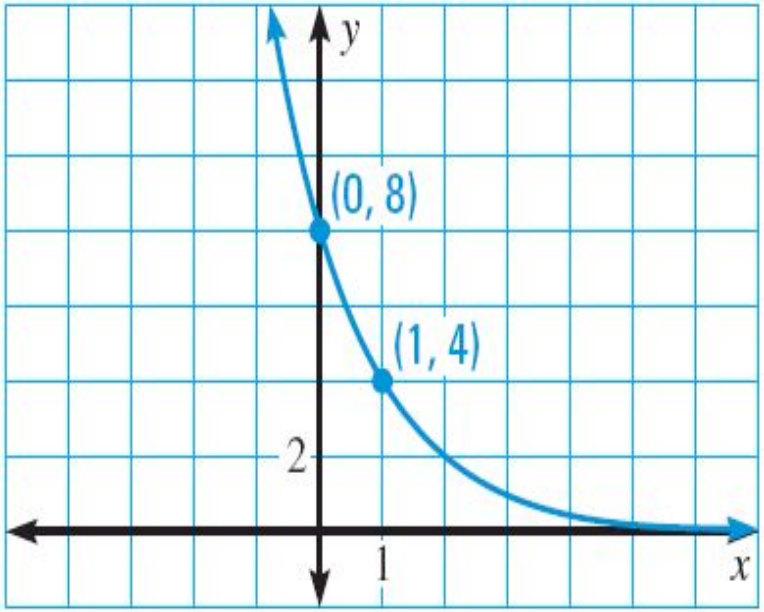
$12 = 10 \cdot b^1$ **Substitute.**

$1.2 = b$ **Solve.**

A function rule is $y = 10(1.2)^x$. This represents exponential growth.

We Do

2. **Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.**



The graph represents exponential decay ($y = ab^x$ where $0 < b < 1$).The y-intercept is 8, so a = 8. Find the value of b by using the point (1, 4) and a = 8.

$y = ab^x$ **Write function.**

$4 = 8 \cdot b^1$ **Substitute.**

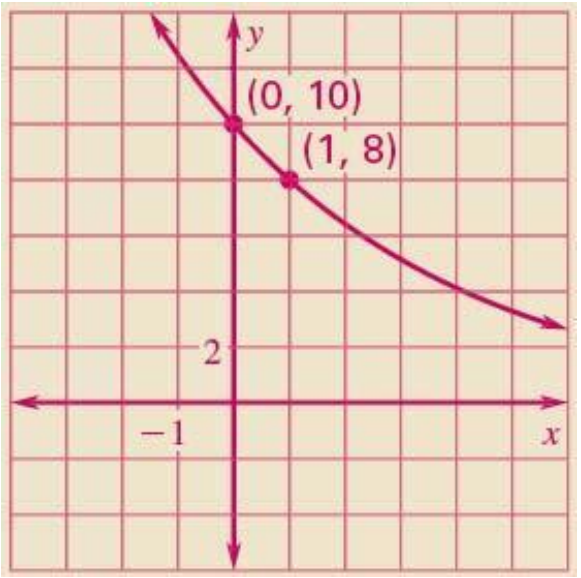
$0.5 = b$ **Solve.**

A function rule is $y = 8(0.5)^x$. This represents exponential decay.

You Do – I Watch

3. **The graph of an exponential function passes through the points (0, 10) and (1, 8). Graph the function. Tell whether the graph represents *exponential growth* or *exponential decay*. Write a rule for the function.**

ANSWER



Steps:

1) Write the formula

2) Find the a (y-int)

3) Substitute the (x, y)

4) Solve for b.

5) Rewrite the rule.

Exponential Growth & Decay Functions

When a quantity grows by a **fixed percent** at regular intervals, the pattern can be represented by the functions:

- **Growth:** $y = a(1 + r)^x$
- **Decay:** $y = a(1 - r)^x$

- a = initial amount before measuring growth/decay
- r = growth/decay rate (often a percent)
- x = number of time intervals that have passed

Compound Interest

$$A = P(1 + r/n)^{nt}$$

$$A = P(1 - r/n)^{nt}$$

- A = the balance
- P = principal initial amount
- r = annual interest rate
- n = number of times interest is compounded per year
- t = time in years

Time intervals: Annual = 1

Semi-annual = 2

Quarterly = 4

Monthly = 12

Example

On January 1, 1999, the price of gasoline was $1.39 per gallon. If the price increased by 0.5% per month, what was the cost of one gallon of gasoline, to the nearest cent, on January 1 one year later?

Example

You owe $4,860 on your discover card. The interest is compounding quarterly starting at 6.99%. What will be your balance next year if you don’t make any payments?

Which is it: Growth or Decay?

Write **Growth** or **Decay**:

1. Making payments on a mortgage loan
2. Credit card accruing interest
3. Savings account earning interest
4. Spread of a virus
5. Value of a collectible over time

Real-World Example

COLLECTOR CAR

The owner of a 1953 Hudson Hornet convertible sold the car at an auction. The owner bought it in 1984 when its value was $11,000. The value of the car increased at a rate of 6.9% per year.

Real-World Problem

a. **Write a function that models the value of the car over time.**

b. **The auction took place in 2004. What was the approximate value of the car at the time of the auction? Round your answer to the nearest dollar.**

SOLUTION

a. **Let C be the value of the car (in dollars), and let t be the time (in years) since 1984. The initial value a is \$11,000, and the growth rate r is 0.069.**

$C = a(1 + r)^t$ **Write exponential growth model.**

$= 11{,}000(1 + 0.069)^t$ **Substitute 11,000 for *a* and 0.069 for *r*.**

$= 11{,}000(1.069)^t$ **Simplify.**

b. To find the value of the car in 2004, 20 years after 1984, substitute 20 for *t*.

$C = 11{,}000(1.069)^{20}$ **Substitute 20 for *t*.**

$\approx 41{,}778$ **Use a calculator.**

ANSWER

In 2004 the value of the car was about $41,778.

FORESTRY

The number of acres of Ponderosa pine forests decreased in the western United States from 1963 to 2002 by 0.5% annually. In 1963 there were about 41 million acres of Ponderosa pine forests.

a. Write a function that models the number of acres of Ponderosa pine forests in the western United States over time.

b. **To the nearest tenth, about how many million acres of Ponderosa pine forests were there in 2002?**

SOLUTION

a. **Let P be the number of acres (in millions), and let t be the time (in years) since 1963. The initial value is 41, and the decay rate is 0.005.**

$P = a(1 - r)^t$ **Write exponential decay model.**

$= 41(1 - 0.005)^t$ **Substitute 41 for a and 0.005 for r.**

$= 41(0.995)^t$ **Simplify.**

b. **To the nearest tenth, about how many million acres of Ponderosa pine forests were there in 2002?**

$P = 41(0.995)^{39} \approx 33.7$

Substitute 39 for *t*. Use a calculator.

ANSWER

There were about 33.7 million acres of Ponderosa pine forests in 2002.

5. **WHAT IF? In Example 5, suppose the decay rate of the forests remains the same beyond 2002. About how many acres will be left in 2010?**

ANSWER

[redacted]

Warm Up

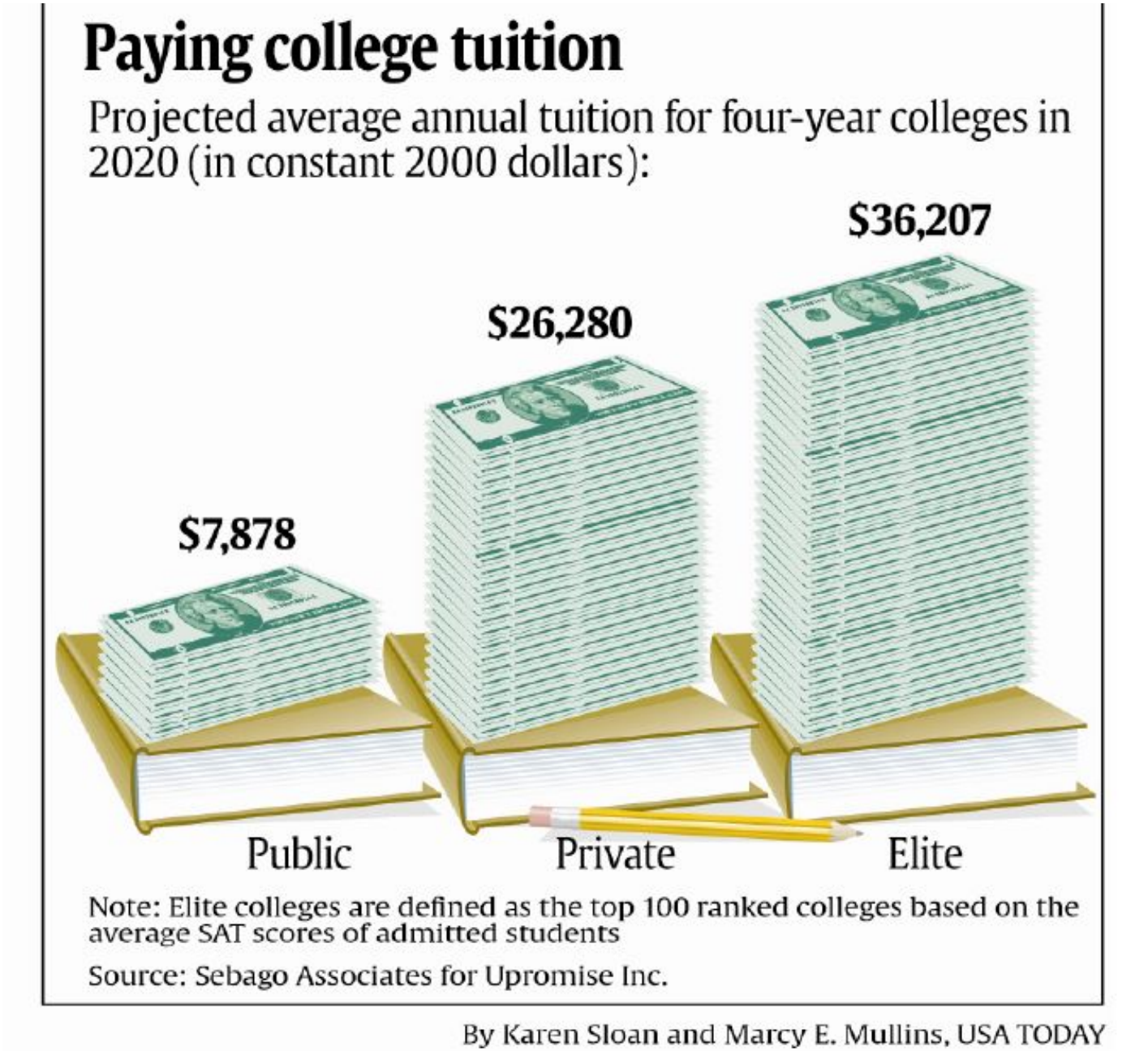
Algebra textbook p. 486
Complete the Lesson Check #1-5
note: #4 is really asking what % of the fish population remains

Classwork:

- *Complete items in text through # 18*
- *Khan Academy:*

Search Exponential Functions and click on the 2nd link

Calculator Investigation



College Discussion

- What are your plans after high school?
- What college or university are you interested in and why?
- Based on the data, do you think college will be affordable to you?
- What economic factors affect the cost of tuition?

Quick Polls

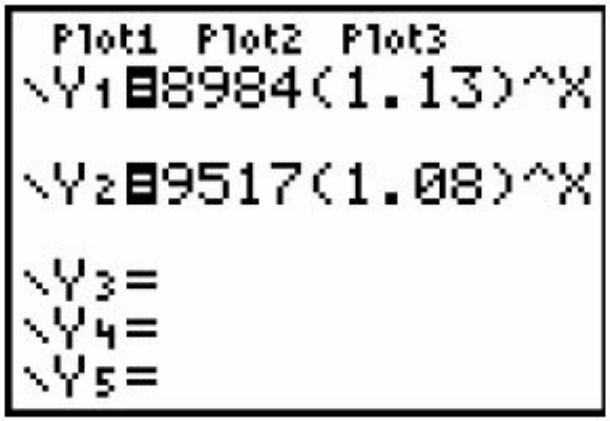
- What is the average tuition cost for in state students?
- What was the average percent of change?

Displaying a Graph

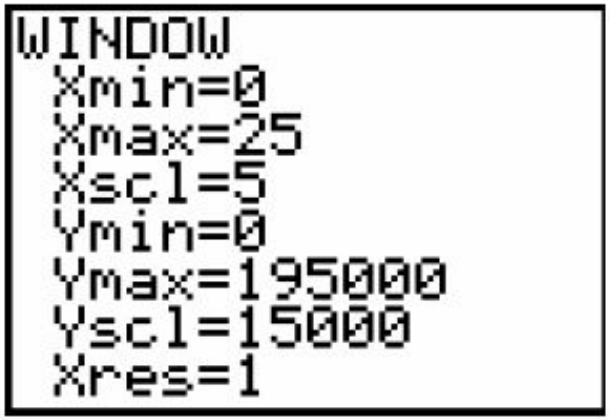
Activity 1

Texas A&M data for in-state expense was used for the first exponential equation, $y=8984(1.13)^x$, and the average cost for in-state was used for the second exponential equation, $y=9517(1.08)^x$.

Press [Y=] and enter both exponential equations listed above in the Y=Editor. If the Y= Editor is not empty, press [CLEAR]. If there are additional entries press [▼][CLEAR] until all are clear. Notice that the = by Y1 and Y2 is highlighted which means it will graph. Pressing [ENTER] when the cursor is on this highlighted area will act as a toggle to turn on or off one or both of the graphs.



Press [WINDOW] and make the follow changes.



“Public Universities Raise Tuition, Fees — and Ire”

- **Q. What is the primary factor for the current increase of college/university costs?**

- **Q. Although most colleges and universities have kept increases to single digits which universities show some of the larger increases?**

- **Q. Why is increased enrollment putting a strain on college and university budgets?**

“Public Universities Raise Tuition, Fees — and Ire”

- **Q. Even with the substantial increases in tuition and fees, why does the article propose that a public college or university education is still a bargain?**

- **Q. What is the University of Kansas doing with a portion of their tuition increase?**

- **Q. What is another factor adding to the increase in college and university enrollments?**

- **Q. What are some of the other ways colleges and universities have increased income without general tuition hikes?**

“Grants More than Offset Soaring University Tuition”

- **Q. Explain what the article means when it states: “For most students, it’s a lot cheaper to go to a four-year public university today than it was just six years ago.”**

- **Q. Why supplement tuition with tax breaks and grants? Why not just charge less for tuition?**